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Engineering Notes

Pressure Waves Generated at the Downstream Corner of a Rectangular Cavity

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Introduction

SELF-SUSTAINING oscillations in cavity flows can be categorized as either fluid dynamic or fluid resonant, depending on the freestream flow velocities. According to Rockwell and Naudascher [1], at a low speed or Mach number, the oscillations are due to flow instability. Studies by Lin and Rockwell [2] and Pereira and Sousa [3] suggested the coupling between the large recirculating vortex with the shear layer provides a possible feedback mechanism. At high Mach numbers, the acoustic waves inside the cavity couple with the vortex shedding from the shear layer to complete the feedback loop. Resonance occurs inside the cavity, and Rossiter [4] was successful in formulating an empirical formula to estimate the frequencies of the various acoustic modes. At transonic and low supersonic Mach numbers, Lee et al. [5] measured mode shapes and phase angles and confirmed the presence of standing waves inside a rectangular cavity. The frequencies of oscillations can be computed with fair accuracies from Rossiter's [4] and Bilanin and Covert's [6] equations, but they cannot predict the dominant acoustic mode and its corresponding amplitude at any instant of time. The difficulties lie in the random nature of the impingement of the vortices at the downstream corner of the cavity. For low-speed flows, Tang and Rockwell [7] showed that three types of interaction are possible, leading to time-dependent pressure amplitudes that are random. In this note, we measured the unsteady pressure in the vicinity of the downstream corner of a rectangular cavity at a low supersonic speed. We computed the pressure waves generated due to vortex impingement, and we present some preliminary results of the amplitude and frequency fluctuations from statistical analyses.

Experiments and Data Reduction

The cavity tested was described in [5]. The length (L) was 3.75 in. The span (W) and the depth (D) were both 0.75 in., giving a ratio of 5 for both L/D and L/W. The cavity was instrumented with 16 fast-response pressure transducers, but only the one located on the downstream wall along the centerline and at a distance of 0.73D from the cavity floor was analyzed. Ideally, we would have liked to have the transducer as close to the corner as possible. However, the wind-tunnel model was originally designed for internal weapons bay investigations, and modifications to relocate the transducer were not possible. The transducer location is shown by the symbol T in the inset of Fig. 1. The frequency response of the transducer was about 20 kHz, and calibration showed a practically flat response up to a test frequency of 6 kHz. Data from the pressure transducer were sampled at 18 kHz for 5 s, following low-pass filtering at 6 kHz, and using an eighth-order Butterworth filter. The low-pass frequency was kept constant and the highest acoustic frequency analyzed was less than 5 kHz. The measured pressure $p(t)$ was expressed in coefficient form as $C_p(t) = [p(t) - p_\infty]/q$, where $q = 1/2\rho U^2$ with ρ and U being the air density and freestream velocity, respectively. The boundary layer ahead of the cavity was tripped to promote a turbulent boundary-layer development at the cavity location. The tests were conducted in the 5 in. trisonic wind tunnel at the Institute for Aerospace Research, National Research Council, Canada at a test Mach number of 1.11. The corresponding Reynolds number was 2.83×10^6 per foot and $q = 10.794$ psi. The descriptions of the model, the test facility, and instrumentation are given in [5].

Joint time-frequency analyses were carried out on the instantaneous C_p time series using a short-time Fourier transform algorithm from the LabVIEW Signal Processing Toolset. The expression for the spectrogram SP(t, f) is

$$\mathrm{SP}\,(t,\, f) = \left| \int_{-\infty}^{\infty} C_p(\tau) w(\tau - t) e^{-i2\pi f \tau}\, \mathrm{d}\tau \right|^2 \qquad (1)$$

where f is the frequency and $w(\tau)$ is a window function. A Hanning window with a length of 256 data points was used in the analysis. This window length was found to give satisfactory qualitative results to illustrate the frequency and amplitude fluctuations phenomena. The program also required the number of frequency bins to be specified, and we chose the bin number to be 256. For the parameters used, the frequency resolution was $\Delta f = 70.31$ Hz and, from the uncertainty principle, the time resolution $\Delta t \geq 1.131$ ms. The number of time steps in the output data was fixed by LabVIEW to be 508 for this case, giving a time step of 6.93 ms.

Results

Figure 1 shows the spectrogram of the pressure measured at transducer T, near the cavity downstream corner, using Eq. (1). For clarity, only 25 equally spaced time steps are included, with a time interval of 138.8 ms. For this Mach number, but at a slightly different Reynolds number and q ($Re = 2.75 \times 10^6$ per foot and $q = 10.8$ psi, respectively), Lee et al. [5] deduced the modal frequencies from the cavity floor pressure measurements. The two dominant modes were close to those determined from a modified form of Rossiter's [4] semi-empirical formula derived by Heller et al. [8]. The measured

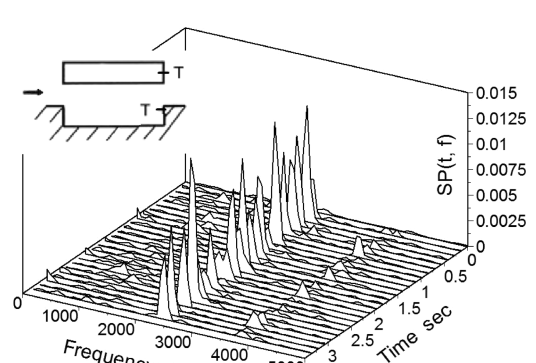

Fig. 1 Spectrogram of pressure fluctuations at downstream cavity corner.

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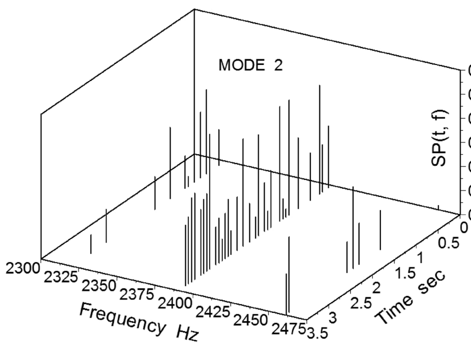

Fig. 2 Variation of peak spectral value with time for mode 2.

values of the Strouhal number St were 0.64 and 1.0 for modes 2 and 3, respectively (corresponding frequencies were 2335 and 3752 Hz, respectively). These values were based on the average frequencies of 13 floor pressure transducers deduced from fast Fourier transform (FFT) computations using 3.527 s of data.

At each time step, we identified the frequency of each mode by the peak in the spectra. This varied within a small range of values and was slightly different from that determined from FFT averaged over 3.527 s. To demonstrate the modal frequency fluctuations, we selected sufficiently large frequency bands about the two Rossiter's [4] frequencies. For modes 2 and 3, we choose $2179 < f < 2601$ Hz and $3585 < f < 4078$ Hz, respectively. At each time step, we determined the maximum $\mathrm{SP}(t, f)$ and the corresponding frequencies inside the two frequency bands. Figure 2 gives a three-dimensional plot for mode 2. For clarity, only 51 time steps are shown, with a time interval of 69.4 ms.

The computations from LabVIEW gave 128 frequencies separated by 70 Hz. For mode 2, the maximum spectral values were found at only three frequencies of 2320, 2390, and 2460 Hz, respectively, inside the selected frequency band. Making the band larger will yield similar results, because observations from Fig. 1 showed the spectral values dropped off rapidly away from the Rossiter's [4] frequencies. Out of the 51 observations shown in Fig. 2, the peak SP occurrences at these three frequencies were 23.5, 60.8, and 15.7%, respectively. Hence, we concluded that $f = 2390$ Hz was the most likely frequency to be detected.

For mode 3, the SP values were much smaller than those for mode 2, as seen in Fig. 1. Figure 3 shows the variations of maximum SP values within a frequency band about the Rossiter's [4] frequency. Maximum SP values were found at five frequencies $f = 3726$, 3796, 3867, 3937, and 4007 Hz, respectively. The corresponding occurrences were 23.5, 41.2, 19.7, 9.8, and 5.8%, for a sample of 51 observations. The results indicated the maximum SP was more likely to be found at $f = 3796$ Hz, whereas $f = 3726$ Hz was the second most likely frequency for which the maximum SP occurred.

The flow near the downstream corner of the cavity was dominated by the impingement of the vortex at the corner. Schlieren photographs taken by Rossiter [4] at high subsonic flow for an $\mathrm{L/D} = 4$ cavity showed the vortices generating oscillations for modes 2 and 3 were of the order of 0.5D in diameter. To completely

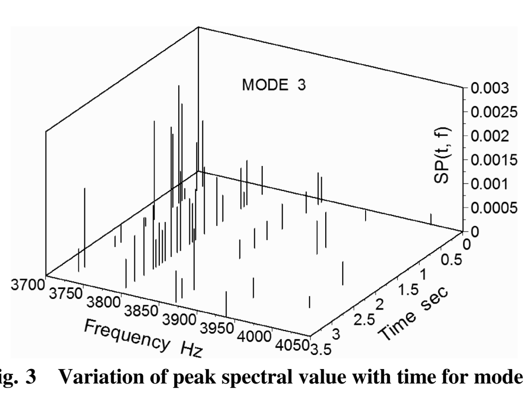

Fig. 3 Variation of peak spectral value with time for mode 3.

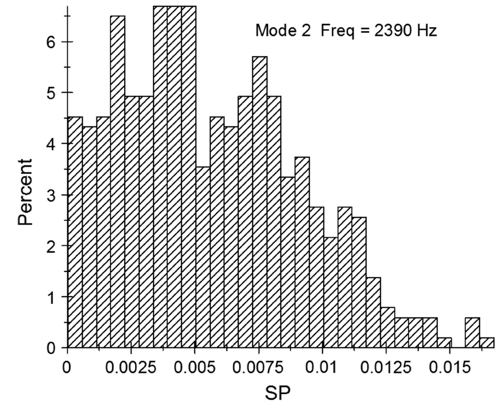

Fig. 4 Histogram for mode 2.

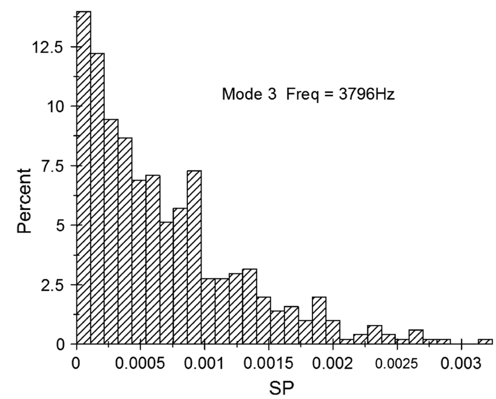

Fig. 5 Histogram for mode 3.

escape undistorted above the rear horizontal surface of the cavity, or be completely captured inside the cavity, required very large motions of the shear layer. Most likely, the vortices were clipped [7] for a large percent of the observations. Some of the vortical flow was swept downward along the cavity rear wall into the recirculating vortex, whereas part of the clipped vortex moved downstream along the cavity rear horizontal surface. In Fig. 1, the magnitude of the spectrogram $\mathrm{SP}(t, f_n)$ $(n = 2, 3)$ shows large changes with time and has a random appearance. For mode 2, histograms were computed from MATLAB® using 30 bins. Figure 4 shows the results at $f = 2390$ Hz, corresponding to the most probable frequency, shown in Fig. 2. The histogram shows the most probable value to be about 0.00375. For mode 3, the histogram given in Fig. 5 shows that values of SP close to zero values have the highest percentage of occurrence. This is also observed in Fig. 1, in which the SP values of mode 3 are close to zero for most of the time.

Conclusions

Pressure measurements near the downstream corner of a rectangular cavity at low supersonic speed showed two oscillating modes to be present. It was found that the modal frequencies varied with time, but the fluctuations were small. Large pressure fluctuations within small frequency bands about the modal frequencies were observed. Histograms were computed, giving the most likely spectral values. The results showed that mode 3 had very small or near-zero amplitudes, and mode 2 was the dominant mode. The small range of frequency for which fluctuations were observed suggested the existing theories of predicting the modal frequencies by assuming them to be time independent could still provide close approximate values when compared with experiments. However, any attempt to predict the modal amplitudes must take into consideration the random pressure fluctuations due to impingement of the downstream traveling vortices with the corner of the cavity.

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